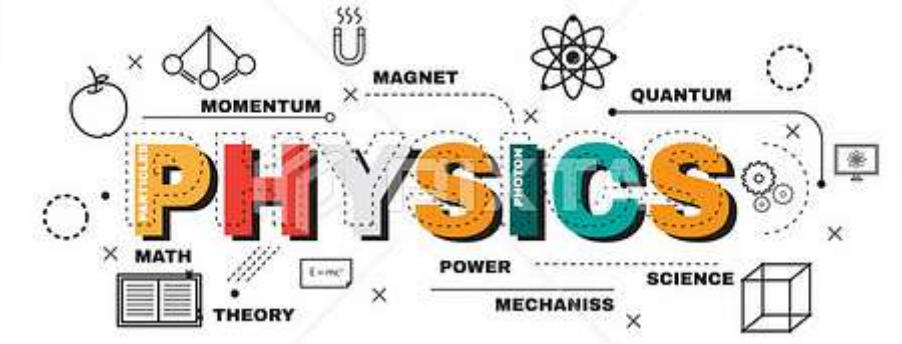


CHAPTER [1]

Units, Physical Quantities & Vectors

[PHYS 101] · General Physics I

Kuwait University

PREPARED BY

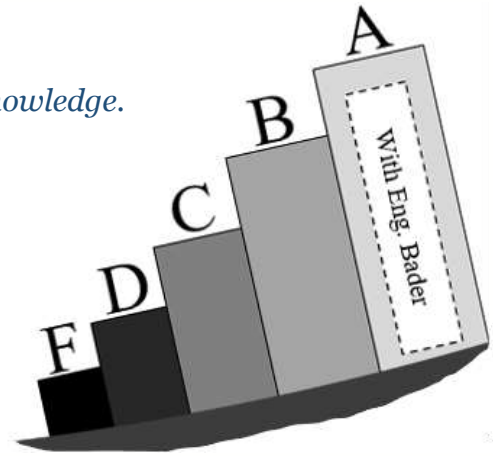
Eng. Bader Al-Enezi

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> *Imagination is more important than knowledge.*
>
> — Albert Einstein





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1) Fundamental Quantities & SI Base Units

Quantity	Symbol	Dimension	SI Base Unit	Common Unit Examples
Length	L, x, y, z, r	[L]	meter (m)	mm, cm, m, km, mile
Time	t	[T]	second (s)	µs, ms, s, min, h, day
Mass	m	[M]	kilogram (kg)	µg, mg, g, kg, ton
Electric Current	I	[I]	ampere (A)	µA, mA, A, kA
Temperature	T	[Θ]	kelvin (K)	K, °C (T + 273.15)
Amount of Substance	n	[N]	mole (mol)	mmol, mol, kmol
Luminous Intensity	Iv	[J]	candela (cd)	mcd, cd, kcd

The 7 SI base units. Chapter 1 (Ch.1) focuses on Length, Time, and Mass — the other four are shown here for a complete reference.

2) Unit Prefixes

Prefix	Symbol	Value	Example
nano	n	10^{-9}	1 ns = 10^{-9} s
micro	µ	10^{-6}	1 µs = 10^{-6} s
milli	m	10^{-3}	1 mm = 10^{-3} m
kilo	k	10^{3}	1 kg = 1000 g
Mega	M	10^{6}	1 M = 10^{6}

3) Unit Consistency & Conversions

- Basic rule: if $A = B$ then $Unit(A) = Unit(B)$ — the units on both sides of any equation must match.
- Common conversions:
- 1 km = 1000 m
- 1 km/h = 1 / 3.6 m/s ⇄ 1 m/s = 3.6 km/h
- 1 kg = 1000 g

4) Vector Basics

- Scalar quantity: has magnitude only — e.g. mass, time, volume, distance, speed.
- Vector quantity: has magnitude AND direction — e.g. displacement, velocity, acceleration, force.
- The magnitude of a vector $|A|$ is always a scalar quantity.
- A = B only if the two vectors have the same magnitude and the same direction.
- Graphical addition uses the head-to-tail method; vectors can be added in any order: $R = A + B + C$
- Vector subtraction: $C = A - B = A + (-B)$

5) Components of a Vector

$Ax = A\cos\theta \qquad Ay = A\sin\theta$

$|A| = \sqrt{(Ax^2 + Ay^2)} \qquad \theta = \tan^{-1}(Ay / Ax)$

(if the angle β is taken from the y-axis: $Ax = A\sin\beta$, $Ay = A\cos\beta$, $\beta = \tan^{-1}(Ax / Ay)$)

6) Unit Vectors

- A unit vector has magnitude 1 with no units: $\hat{\imath}$ along +x, $\hat{\jmath}$ along +y, $\hat{k}$ along +z.

$A = Ax\,\hat{\imath} + Ay\,\hat{\jmath} + Az\,\hat{k}$

$A + B = (Ax+Bx)\,\hat{\imath} + (Ay+By)\,\hat{\jmath} + (Az+Bz)\,\hat{k}$

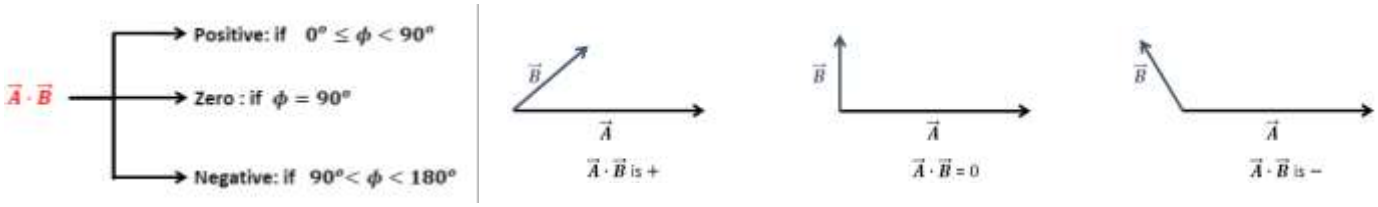
$A - B = (Ax-Bx)\,\hat{\imath} + (Ay-By)\,\hat{\jmath} + (Az-Bz)\,\hat{k}$

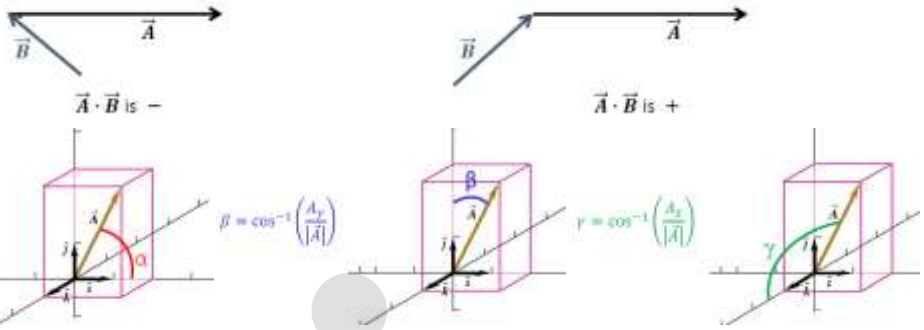
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7) Dot (Scalar) Product

$A \cdot B = |A||B| \cos\varphi = AxBx + AyBy + AzBz \quad \rightarrow$ *result is a scalar*

- Positive if $0° \leq \varphi < 90°$ — zero if $\varphi = 90°$ — negative if $90° < \varphi < 180°$
- Commutative: $A \cdot B = B \cdot A$
- Direction (cosine) angles: $\alpha = \cos^{-1}(Ax/A)$, $\beta = \cos^{-1}(Ay/A)$, $\gamma = \cos^{-1}(Az/A)$





Direction cosine angles α, β, γ measured from the +x, +y, +z axes

8) Cross (Vector) Product

$A \times B = C = A\,B \sin\varphi$ *(direction given by the right-hand rule)*

$A \times B = \hat{\imath}(AyBz - AzBy) - \hat{\jmath}(AxBz - AzBx) + \hat{k}(AxBy - AyBx)$

- If $\varphi = 0°$ then $A \times B = 0$, so: $\hat{\imath}\times\hat{\imath} = \hat{\jmath}\times\hat{\jmath} = \hat{k}\times\hat{k} = 0$
- $\hat{\imath}\times\hat{\jmath} = \hat{k}$, $\hat{\jmath}\times\hat{k} = \hat{\imath}$, $\hat{k}\times\hat{\imath} = \hat{\jmath}$ / $\hat{\jmath}\times\hat{\imath} = -\hat{k}$, $\hat{k}\times\hat{\jmath} = -\hat{\imath}$, $\hat{\imath}\times\hat{k} = -\hat{\jmath}$

Notes:

(1) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

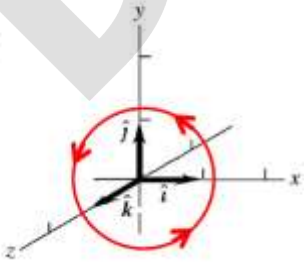
(2) If $\phi = 0°$ then $\vec{A} \times \vec{B} = 0$ $\quad (\vec{A} \times \vec{A} = \hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0)$

(3)

$\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\quad$ $\hat{\jmath} \times \hat{\imath} = -\hat{k}$

$\hat{\jmath} \times \hat{k} = \hat{\imath}$ $\quad$ $\hat{k} \times \hat{\jmath} = -\hat{\imath}$

$\hat{k} \times \hat{\imath} = \hat{\jmath}$ $\quad$ $\hat{\imath} \times \hat{k} = -\hat{\jmath}$



If we have the two vectors:

$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$

$\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$

Then

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}\,(A_y\,B_z - A_z\,B_y) - \hat{\jmath}\,(A_x\,B_z - A_z\,B_x) + \hat{k}\,(A_x\,B_y - A_y\,B_x)$$

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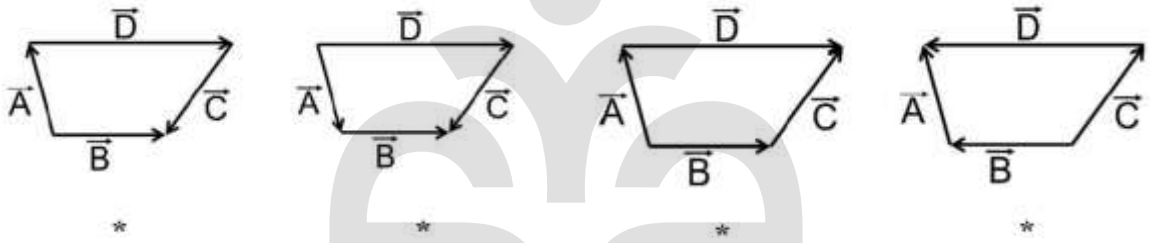
Ch1: Units, Physical Quantities, and Vectors

Slides Questions Chapter 1

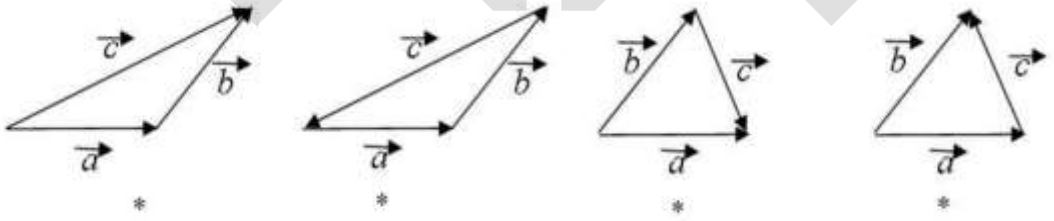
Q1: The velocity v_x of a car at any time is given by the equation $v_x = 60 - 0.4t^2$, where v_x is in m/s and t is in s. What is the unit of the constant 0.4 in this equation?

Q2: The position of a moving object is given by the equation $x = vt + \frac{1}{2}\beta t^2$, where x is given in meters and t is given in seconds. What is the unit of β?

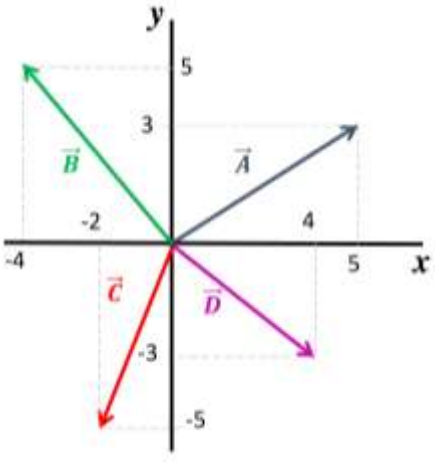
Q3: Which of the following figures satisfies the relation **A** = **B** + **C** − **D** ?



Q4: The vectors **a**, **b**, and **c** are related by **c** = **a** − **b**. Which diagram below illustrates this relationship?



Q5: Express each of the vectors **A**, **B**, **C**, and **D** shown in the figure in unit vector (î, ĵ) notation.

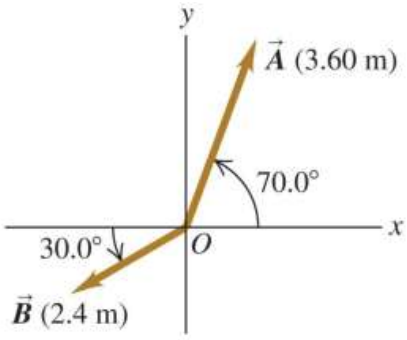


Q6: Vector **A** has a magnitude of 3.60 m and makes an angle of 70.0° with the +x-axis. Vector **B** has a magnitude of 2.4 m and makes an angle of 30.0° with the −x-axis, as shown in the figure. Find:

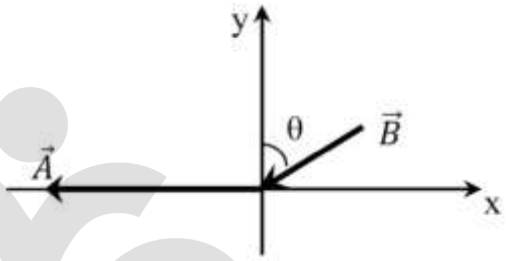
(a) Each vector in terms of the unit vectors $\hat{i}$ and $\hat{j}$.

(b) Vector **C**, using unit vectors, where $\mathbf{C} = 3\mathbf{A} - 4\mathbf{B}$.

(c) The magnitude and direction of **C**.



Q7: Vector **A** and vector **B** are shown in the figure, where θ is the angle between **B** and the +y-axis. $\mathbf{A} \cdot \mathbf{B}$ equals:



(a) AB cos θ

(b) AB sin θ

(c) AB cos(π/2 + θ)

(d) AB sin(π/2 + θ)

Q8: If $\mathbf{A} = a\hat{i} + a\hat{j} + b\hat{k}$, where a and b are constants, and the angle between **A** and the +x-axis is 60°, what is the angle β between **A** and the +y-axis?

(a) 30°

(b) 60°

(c) 90°

(d) 60° < β < 90°

Q9: If $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ and θ is the angle between **A** and **B**, then $\mathbf{C} \cdot \mathbf{A}$ equals:

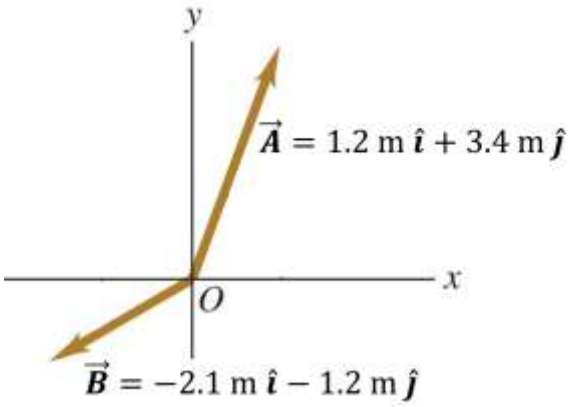
(a) Zero

(b) 1

(c) AC sin θ

(d) AC cos θ

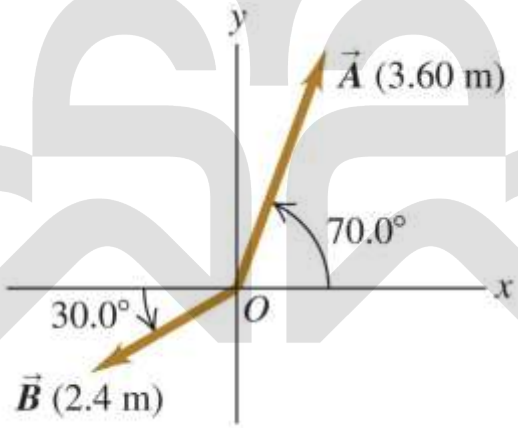
Q10: Given the vectors **A** = 1.2î + 3.4ĵ (m) and **B** = −2.1î − 1.2ĵ (m), as shown in the figure, find:



(a) The scalar (dot) product **A** · **B**.

(b) The magnitude and direction of the vector (cross) product **A** × **B**.

Q10: Given the vectors as shown in the figure, find:



(a) The scalar (dot) product **A** · **B**.

(b) The magnitude and direction of the vector (cross) product **A** × **B**.

Question 1

$v_x = 60 - 0.4\,t^2$ (v_x in m/s, t in s). Find the unit of 0.4.

$[0.4] \times s^2 = m/s$

$[0.4] = (m/s) / s^2 = m/s^3$

✓ **Answer: 0.4 has units of m/s^3.**

Question 2

$x = v\,t + (1/2)\,\beta\,t^2$ (x in m, t in s). Find the unit of β.

$[\beta] \times s^2 = m$

$[\beta] = m / s^2$

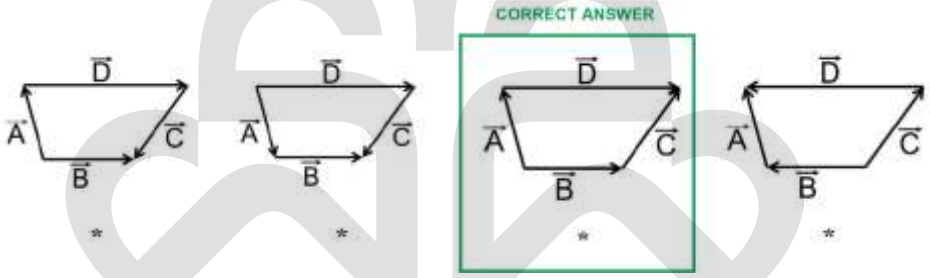
✓ **Answer: β has units of m/s^2.**

Question 3

Which figure satisfies $A = B + C - D$?

$A = B + C - D \rightarrow A + D = B + C$

(tip-to-tail: A then D must end where B then C ends)



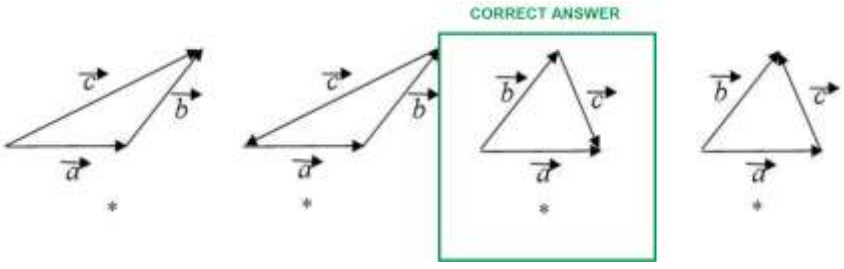
✓ **Answer: 3rd diagram (boxed).**

Question 4

c = a − b. Which diagram illustrates this?

$c = a - b \rightarrow a = b + c$

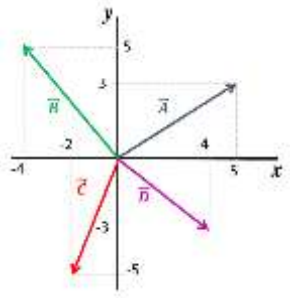
(tip-to-tail: b then c must end where a ends)



✓ **Answer:** **3rd diagram (boxed).**

Question 5

Express A, B, C, D in unit-vector (î, ĵ) notation.



A = 5 î + 3 ĵ

B = – 4 î + 5 ĵ

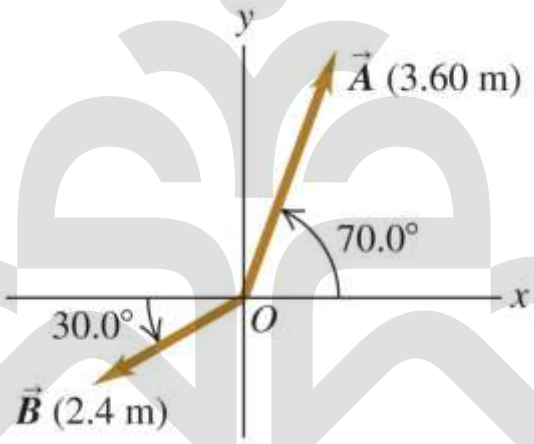
C = – 2 î – 5 ĵ

D = 4 î – 5 ĵ

✓ **Answer:** **A = 5i+3j, B = -4i+5j, C = -2i-5j, D = 4i-5j**

Question 6

A = 3.60 m at 70.0° from +x. B = 2.4 m at 30.0° from − x. Find A, B (unit vectors); C = 3A − 4B; |C| and direction.



A_x = 3.60 cos70.0° = 1.23 m A_y = 3.60 sin70.0° = 3.38 m

A = 1.23 î + 3.38 ĵ (m)

B direction = 210.0° from +x (180° + 30°)

B_x = 2.4 cos210.0° = − 2.08 m B_y = 2.4 sin210.0° = − 1.20 m

B = – 2.08 î – 1.20 ĵ (m)

3A = 3.69 î + 10.15 ĵ 4B = − 8.31 î − 4.80 ĵ

C = 3A – 4B = 12.0 î + 14.9 ĵ (m)

$|C| = \sqrt{(12.0^2 + 14.9^2)} = \sqrt{(367.6)} \approx 19.2$ m

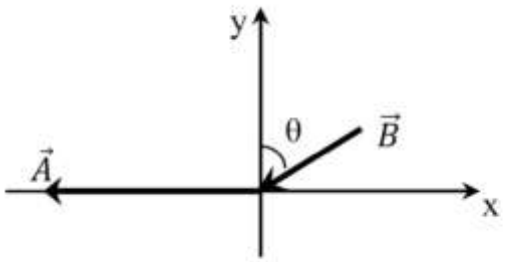
$\theta = tan^{-1}(14.9 / 12.0) \approx 51.2°$ (above +x)

✓ **Answer:** **A=1.23i+3.38j m, B=-2.08i-1.20j m, C=12.0i+14.9j m, |C|=19.2 m at 51.2 deg.**

Question 7

θ = angle between B and +y-axis. A · B = ?

(a) AB cosϑ (b) AB sinϑ (c) AB cos(π/2 + ϑ) (d) AB sin(π/2 + ϑ)



A: 180° B: (90° − θ) (from +x-axis)

angle(A,B) = 180° − (90° − θ) = π/2 + θ

A · B = AB cos(π/2 + θ)

✓ **Answer: (c) AB cos(pi/2 + theta)**

Question 8

A = a î + a ĵ + b k̂. Angle(A,+x) = 60°. Find β = angle(A,+y).

(a) 30° (b) 60° (c) 90° (d) 60° < θ < 90°

|A| = √(a²+a²+b²) = √(2a²+b²)

cos60° = a / |A| = cos(β) (Ax = Ay = a)

β = 60°

✓ **Answer: (b) 60 degrees**

Question 9

C = A × B. θ = angle(A,B). C · A = ?

(a) Zero (b) 1 (c) AC sinϑ (d) AC cosϑ

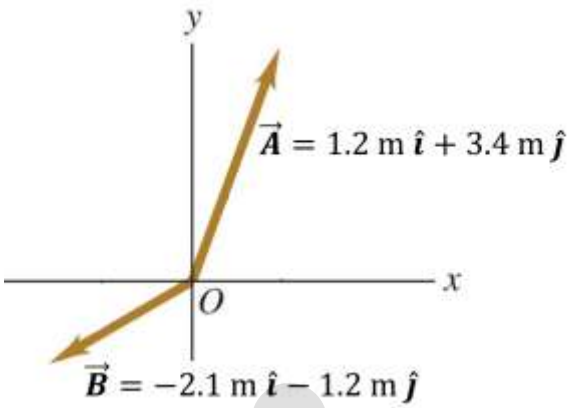
C = A × B → C ⊥ A → angle(C,A) = 90°

C · A = |C||A| cos90° = 0

✓ **Answer: (a) Zero**

Question 10

A = 1.2 î + 3.4 ĵ (m), B = − 2.1 î − 1.2 ĵ (m). Find A · B and A × B.



A · B = (1.2)(− 2.1) + (3.4)(− 1.2) = − 2.52 − 4.08

A · B = – 6.60 m²

A × B = (Ax By − Ay Bx) k̂ = [(1.2)(− 1.2) − (3.4)(− 2.1)] k̂ = [− 1.44 + 7.14] k̂

A × B = 5.70 k̂ m² (+z, out of page)

✓ **Answer: A.B = -6.60 m^2. A x B = 5.70 m^2 along +k (out of the page).**

--- End of solutions ---

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Ch1: Units, Physical Quantities, and Vectors

Book Questions Chapter 1

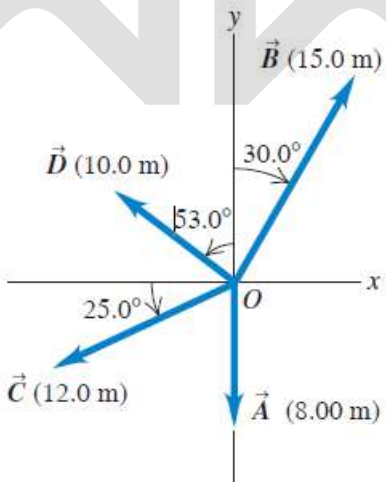
Exercises

1.24: Let θ be the angle that the vector **A** makes with the +x-axis, measured counterclockwise from that axis. Find angle θ for a vector that has these components: (a) Ax = 2.00 m, Ay = −1.00 m; (b) Ax = 2.00 m, Ay = 1.00 m; (c) Ax = −2.00 m, Ay = 1.00 m; (d) Ax = −2.00 m, Ay = −1.00 m.

1.26: Vector **A** is in the direction 40.0° clockwise from the −y-axis. The x-component of **A** is Ax = −20.0 m. (a) What is the y-component of **A**? (b) What is the magnitude of **A**?

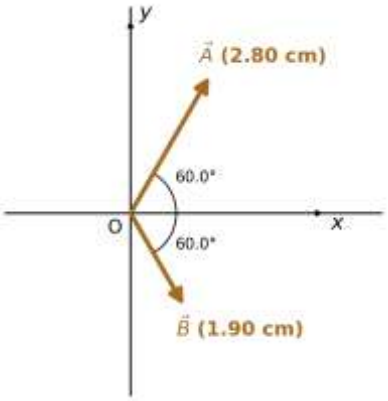
1.29: For the vectors **A** and **B** in Fig. E1.22, use the method of components to find the magnitude and direction of (a) the vector sum **A** + **B**; (b) the vector sum **B** + **A**; (c) the vector difference **A** − **B**; (d) the vector difference **B** − **A**.

Figure E1.22



1.38: You are given two vectors **A** = −3.00î + 6.00ĵ and **B** = 7.00î − 2.00ĵ. Let counterclockwise angles be positive. (a) What angle does **A** make with the +x-axis? (b) What angle does **B** make with the +x-axis? (c) Vector **C** is the sum of **A** and **B**, so **C** = **A** + **B**. What angle does **C** make with the +x-axis?

1.44: For the two vectors in Fig. E1.33, find the magnitude and direction of (a) the vector product **A** × **B**; (b) the vector product **B** × **A**.



1.67: You are lost at night in a large, open field. Your GPS tells you that you are 122.0 m from your car, in a direction 58.0° east of south. You walk 72.0 m due west along a ditch. How much farther, and in what direction, must you walk to reach your car?

1.81: Vector **A** has magnitude 12.0 m, and vector **B** has magnitude 12.0 m. The scalar product **A** · **B** is 76.0 m^2. What is the magnitude of the vector product between these two vectors?

Questions

Q1.10: Is the vector ($\hat{i} + \hat{j} + \hat{k}$) a unit vector? Is the vector ($3.0\hat{i} - 2.0\hat{j}$) a unit vector? Justify your answers.

Q1.15: Can you find a vector quantity that has a magnitude of zero but components that are not zero? Explain. Can the magnitude of a vector be less than the magnitude of any of its components? Explain.

Q1.18: If **A** and **B** are nonzero vectors, is it possible for both **A** · **B** and **A** × **B** to be zero? Explain.

Q1.24: Show that, no matter what **A** and **B** are, **A** · (**A** × **B**) = 0. (Hint: Do not look for an elaborate mathematical proof. Consider the definition of the direction of the cross product.)

Q1.25: (a) If **A** · **B** = 0, does it necessarily follow that A = 0 or B = 0? Explain. (b) If **A** × **B** = 0, does it necessarily follow that A = 0 or B = 0? Explain.

Exercises

1.24

Find θ (CCW from +x-axis) for:

(a) A_x=2.00, A_y=−1.00 (b) A_x=2.00, A_y=1.00 (c) A_x=−2.00, A_y=1.00 (d) A_x=−2.00, A_y=−1.00 (m)

reference angle = $tan^{-1}(\frac{Ay}{Ax})$ = $tan^{-1}(\frac{-1}{2})$ = −26.6°=360° −26.6°=333.4°

(a) Q4 (Ax>0,Ay<0): θ = 333.4°

(b) Q1 (Ax>0,Ay>0): θ = 26.6°

(c) Q2 (Ax<0,Ay>0): θ = 180° − 26.6° = 153.4°

(d) Q3 (Ax<0,Ay<0): θ = 180° + 26.6° = 206.6°

✓ **Answer: (a) 333.4° (b) 26.6° (c) 153.4° (d) 206.6°**

1.26

A is 40.0° clockwise from the − y-axis. Ax = − 20.0 m. Find Ay and |A|.

direction from +x-axis: 270° − 40.0° = 230.0°

Ax = |A| cos230.0° → |A| = (− 20.0) / cos230.0° = (− 20.0)/(− 0.6428)

|A| = 31.1 m

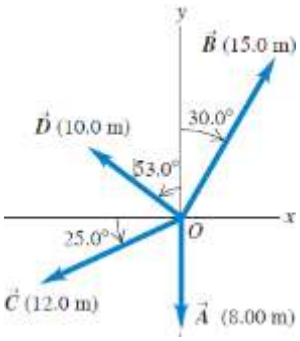
Ay = |A| sin230.0° = 31.1 × (− 0.7660)

Ay = − 23.8 m

✓ **Answer: Ay = -23.8 m, |A| = 31.1 m**

1.29

Using Fig. E1.22, find magnitude and direction of (a) A+B (b) B+A
(c) A− B (d) B− A.



A = 8.00 m at 270.0° → A = (0.00, − 8.00) m

B = 15.0 m at 60.0° → B = (7.50, 13.0) m (Bx=15.0cos60°, By=15.0sin60°)

(a),(b) A+B = B+A = (0.00+7.50, − 8.00+13.0) = (7.50, 4.99) m

|A+B| = $\sqrt{(7.50^2 + 4.99^2)}$ = $\sqrt{(81.2)}$ = 9.01 m θ =$tan^{-1}(\frac{4.99}{7.50})$ = 33.6°

(c) A− B = (0.00− 7.50, − 8.00− 13.0) = (− 7.50, − 21.0) m

|A− B| =$\sqrt{(7.50^2 + 21.0^2)}$ = $\sqrt{(496.8)}$ = 22.3 m θ = 180°+tan^{-}(21.0/7.50) = 250.3°

(d) B− A = − (A− B) = (7.50, 21.0) m |B− A| = 22.3 m θ = 70.3°

✓ **Answer: (a),(b) 9.01 m at 33.6 deg (c) 22.3 m at 250.3 deg (d) 22.3 m at 70.3 deg**

1.38

A = − 3.00 î + 6.00 ĵ, B = 7.00 î − 2.00 ĵ. Find angle of A, angle of B, and angle of C = A+B.

A: Q2, reference = arctan(6.00/3.00) = 63.4° → θ of A = 180° − 63.4° = 116.6°

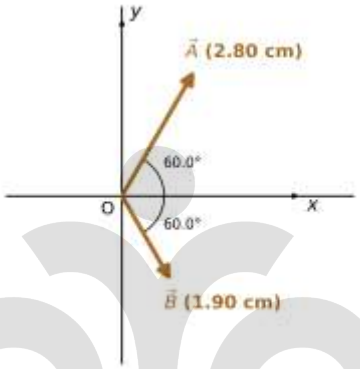
B: Q4, reference = arctan(2.00/7.00) = 16.0° → θ of B = − 16.0° (= 344.0°)

C = A+B = (− 3.00+7.00, 6.00− 2.00) = (4.00, 4.00) → θ of C = arctan(4.00/4.00) = 45.0°

✓ **Answer: angle(A) = 116.6 deg, angle(B) = -16.0 deg, angle(C) = 45.0 deg**

1.44

Using Fig. E1.33, find magnitude and direction of (a) A × B (b) B × A.



angle between A and B: θ = 60.0° + 60.0° = 120.0°

|A × B| = AB sinθ = (2.80)(1.90) sin120.0° = 5.32 × 0.8660

|A × B| = |B × A| = 4.61 cm²

(right-hand rule: A rotated into B is clockwise, so A x B points into the page; B x A points out of the page)

✓ **Answer: (a) A x B = 4.61 cm^2, into the page (-z) (b) B x A = 4.61 cm^2, out of the page (+z)**

1.67

Car is 122.0 m away, 58.0° east of south. You walk 72.0 m due west. Find the remaining distance and direction to the car.

car position (east, north) = (122.0 sin58.0°, − 122.0 cos58.0°) = (103.5, − 64.7) m

your position after walking west 72.0 m = (− 72.0, 0.0) m

remaining vector = car − you = (103.5−(− 72.0), − 64.7− 0.0) = (175.5, − 64.7) m

distance = √(175.5² + 64.7²) = √(34970) = 187.0 m

direction = arctan(64.7 / 175.5) = 20.2° south of east

✓ **Answer: 187 m, at 20.2 deg south of east**

1.81

$|A| = 12.0$ m, $|B| = 12.0$ m, $A \cdot B = 76.0$ m^2. Find $|A \times B|$.

$(A \cdot B)^2 + |A \times B|^2 = (|A||B|)^2$ (since $\cos^2\theta+\sin^2\theta=1$)

$|A \times B| = \sqrt{((|A||B|)^2 - (A \cdot B)^2)} = \sqrt{((12.0 \times 12.0)^2 - 76.0^2)} = \sqrt{(144.0^2 - 76.0^2)}$

$= \sqrt{(20736 - 5776)} = \sqrt{(14960)}$

✓ **Answer: |A x B| = 122 m^2**

Questions

Q1.10

Is $(\hat{i} + \hat{j} + \hat{k})$ a unit vector? Is $(3.0\hat{i} - 2.0\hat{j})$ a unit vector? Justify.

$|\hat{i}+\hat{j}+\hat{k}| = \sqrt{(1^2+1^2+1^2)} = \sqrt{(3)} \approx 1.73$ – not 1

$|3.0\hat{i} - 2.0\hat{j}| = \sqrt{(3.0^2+2.0^2)} = \sqrt{(13)} \approx 3.61$ – not 1

✓ **Answer: Neither is a unit vector - a unit vector must have magnitude exactly 1, and both magnitudes above are not 1.**

Q1.15

Can a vector have zero magnitude but nonzero components? Can the magnitude be less than the magnitude of any of its components?

$|A| = \sqrt{(Ax^2+Ay^2+Az^2)} = 0 \rightarrow Ax=Ay=Az=0$ (a sum of squares is 0 only if every term is 0)

$|A| = \sqrt{(Ax^2+Ay^2+Az^2)} \geq \sqrt{(Ax^2)} = |Ax|$ for every component

✓ **Answer: No, and no - zero magnitude forces every component to be zero, and |A| is always >= the magnitude of any single component.**

Q1.18

If A and B are nonzero, is it possible for both $A \cdot B$ and $A \times B$ to be zero?

$A \cdot B = 0 \rightarrow \theta = 90°$ $A \times B = 0 \rightarrow \theta = 0°$ or $180°$

✓ **Answer: No - theta cannot be 90 deg and 0/180 deg at the same time unless A or B is zero.**

Q1.24

Show that $A \cdot (A \times B) = 0$, for any A and B.

$A \times B$ is perpendicular to A (definition of the cross product) $\rightarrow$ angle(A, A× B) = 90°

$A \cdot (A \times B) = |A| \, |A \times B| \cos 90°$

✓ **Answer: A . (A x B) = 0**

Q1.25

(a) If $A \cdot B = 0$, does $A=0$ or $B=0$ necessarily? (b) If $A \times B = 0$, does $A=0$ or $B=0$ necessarily?

(a) $A \cdot B = AB\cos\theta = 0$ also whenever $\theta = 90°$ (e.g. $A=\hat{i}$, $B=\hat{j}$: $A\cdot B=0$, neither is 0)

(b) $A \times B = AB\sin\theta = 0$ also whenever $\theta = 0°$ or $180°$ (e.g. $A=\hat{i}$, $B=2\hat{i}$: $A\times B=0$, neither is 0)

✓ **Answer: No to both - each condition can also occur whenever A and B are perpendicular (a) or parallel/antiparallel (b).**

--- End of solutions ---